

***Household Size, Economies of Scale and Public Goods in
Consumption: A Proposal to resolve the Food Share “Paradox”***

Feridoon Koohi-Kamali 2014*

feridoon.koohi@emory.edu, Department of Economics, Emory University
1602 Fishburne Dr., Rich Building, Atlanta, GA, 30322, USA

Abstract

This paper is addressed to an explanation of the food share paradox encountered in (Deaton and Paxson 1998), and suggests a proposal to resolve it. The paper examines the effect of family size on the demand for food, a commodity group believed to be particularly responsive to changes in economies available to households. It argues that the size effect on the budget share of food is negative; this has only become a paradox because the sources of economies are assumed to be confined to mainly family public goods. The paradox is likely to disappear once within food group private sources of economies are acknowledged.

I. Economies in consumption and the budget share of food

It seems obvious that the size of a household should be a major influence on its standard of living, yet how that influence manifests itself is far from clear. (Deaton and Paxson 1998)-henceforth D-P, is a prominent study investigating this question. They have proposed a model of the size effect on the food share based on a suggestion, due to Dreze, that the economies of scale effect is the result of a relative fall in the price of non-food family public goods as size expands¹. This approach requires the existence of a private good,

* I wish to express my special thanks to John Muellbauer for his insights and valuable suggestions. I am also very grateful to Angus Deaton for his comments. None are responsible for the views expressed here; all remaining errors are entirely mine.

¹A description of this method can be found in (Deaton 1997, 264-8).

which displays a limited substitution effect with respect to a fall in the relative price of public goods resulting from an increase in size. Aggregate food is commonly taken as such a commodity. It has few substitutes outside the group, if not for the wealthy, then at least for poor households in developing economies, where average food consumption is closer to subsistence. Thus, with income per head constant, a fall in the price of public goods, following an increase in size, will release more income, which, with a fall in food consumption due to substitution being relatively small, will result mainly in an income effect on food, increasing available per capita food expenditure. Thus, if the size effect were to influence food expenditure in this manner, one would expect a *positive* correlation between the food share in household budget and household size. This suggests that regressing the food share on per capita income and an independent size variable, should produce a positive coefficient estimate for size, that is, the food share curve of the larger household will be above that of the smaller one². They admit the possibility of other sources, including some scope for substitution effects, but dismiss them as likely to be empirically small.

The implementation of this approach in D-P has resulted in a paradox of strongly *negative* size effects observed in practice³. That is, their evidence

²An exception may occur if there are fixed costs to household public goods consumption, the most important being that for housing. If the fixed cost is already met as size expands, the predicted effect is increase in per head food demand; there is no problem in this case. However, if the size expansion makes it more likely for the household to incur the fixed cost, then economies of scale, and hence per capita food expenditure are reduced. Evidence in D-P shows that in the move from one-person to two-person households, this counter effect is of some importance in developed, but not developing countries, presumably because housing is a relatively less important expenditure category for the latter.

³D-P is a large scale study on consumption effects of size, containing parametric and non-parametric evidence from seven budget surveys of developed (US, UK, and

suggests that per capita food expenditure falls with household size for every one of the developed and developing countries in their study, in sharp contrast to the predicted positive size effect by their model. Moreover, the negative size effect is stronger for developing countries, which ‘deepens’ the paradox, given that their food consumption is closer to subsistence levels, and they have larger average budget shares of food. In the next section, I attempt to demonstrate below that a positive size effect, *not* a negative one, is the expected outcome.

II. An alternative explanation of D-P outcomes.

Not all economies of scale effects are generated through consumption of public goods. There are two other notions of the economies generated through non-shared private goods: that based on household technology and that on discount on bulk purchases; see also David (1962) for the distinction between the two. The private sources of the scale are from within the group and reduce aggregate food expenditure. Let us examine the case of bulk discount for food, likely to be a more prominent source of private economies in consumption⁴. The price paid in this case is a function of food quantity

France) and developing (Taiwan, Thailand, by urban and rural separately, Pakistan, and South Africa (blacks only)) countries. The parametric evidence, also backed by non-parametric results, is based on a food demand function with the same demographic specification as (1), consistently producing a negative sign for size in *all* their results, in exactly the opposite direction to that predicted by the public goods approach.

⁴The similar effect can result from a reduction in food wastage as size expands. Another source is due to return to scale from household technology. This notion is based on the idea that income rises less than proportionately with household size as additional members supply less labour time to the market. *If* this were the outcome, the household’s ratio of non-market time to the quantity of goods purchased would rise. Larger households would thus tend to utilize their relatively cheaper household input, namely non-market time, to produce goods below their market prices, e.g. cooking, so

purchased, so by definition, $q_1 \geq q_2$ if $p_{q_1} \leq p_{q_2} \therefore p_{q_1} q_1 \leq p_{q_2} q_2$, i.e. expenditure on the food declines with bulk discount⁵. If this private source of the scale were the dominant one, given constant PCE, the food share would decline as size expands, in conformity with the Engel law and contrary to the outcome predicted if public goods were the only source. This would then provide an alternative prediction of a negative effect as the expected direction of the household size influence on the food share⁶.

Still, even if the private source of the scale were subordinate to public goods, its presence will modify the scope from the latter. The absence of substitution between food and non-food groups does not necessarily imply that the only change in food expenditure comes from public goods outside the food group. To reach that conclusion requires information not provided by the public goods model. Since it does not appear possible to source separately

also reducing food expenditure. However, D-P point out, it is hard to examine this effect if one has to condition food expenditure solely on total expenditure because time data is unavailable.

⁵(Prais and Houthakkar 1955) general measure of economies of scale is obtained from estimation of a quality effect equation (see note below); one that suggests, with constant per capita expenditure, the price per unit increases with household size. There is also empirical evidence from developing countries that the amount paid per unit increases for households that buy more, usually the better off households, see for example (Koohi-Kamali 2005). However, much of this is probably quality rather not price effects.

⁶Such a possibility is mentioned in D-P that cites a study on the French food market, which supports such a dominant role for bulk discount; and in addition, some of its own estimates for France provide support for this conclusion. However, the more common evidence in D-P for other countries shows the French evidence on the prominent role of bulk discount effect is not typical, so this alternative is likely to have a limited scope as an explanation of the negative size outcome.

components of the measured scale effect, the coefficient estimate on the logarithm of household size in the model of economies in consumption represents a compound parameter value⁷. In general, there are therefore likely to be two changes to food expenditure from an expansion in size: an indirect, spill over effect from outside the food group, and a direct effect from within it. Evidence on the subordinate role of bulk discount does not resolve the identification problem since it does not follow that that effect is necessarily negligible, suggesting the public goods measure of economies underestimates the true (public) size effect⁸.

The identification issue can be more clearly explained in terms of the Prais-Houthakker (P-H) model. (Prais and Houthakker 1955, ch. 10) first proposed and examined the hypothesis that an increase in household size leads to an increase in economies of scale in consumption available to the household, without restricting the sources of economies to either public or private goods. The Engel equivalence scale model can be regarded as a special case of that model, with identical scales across goods, thus resolving the problem of the lack of identification between the commodity-specific scales

⁷(Nelson 1988, esp. table III discussion) proposes a measure of economies of scale based on the product of $m_i = a_i(n_h) \cdot b_i(n_h)$ for each commodity i for household h of size n in a Barten-type model. If i is purely private, $a_i(n_h) = n_h^{1-\theta_i}$ and $\theta_i < 1$; if purely public $b_i(n_h) = n_h^{\theta_i}$; $0 < \theta_i < 1$. When the scale has both private and public sources, additional identifying assumptions are required to separate the effects of the two sources on the scale estimate.

⁸(Pollak and Wales 1981) have pointed out that *private* goods economies of scale “do not involve demographics at all”. If so, their scope could be measured in physical quantities. There is, however, no model, which would allow isolating private economies by a quantity-based measure in order to identify the size-based measure of public scale.

and the general income scale⁹. However, if one were willing to make an identifying assumption, for instance, that the food and income scales are identical, or that they are identical across all commodities, as in the Engel model, then the D-P paradox of a negative size effect disappears, as I demonstrate below. Therefore, it is important to emphasize that the explanation of a negative size effect on the food share is the consequence of making such an identifying assumption. In order to see this, let us consider

⁹More specifically, this method is an extension of their equivalence scale model, relating commodity-specific scales to a general scale. In its simple form, $\frac{p_i q_i}{n^{\theta_i}} = f_i\left(\frac{x}{n^{\theta_0}}\right)$ is the i th commodity Engel curve. $(1-\theta_i)$ measures economies for i , $(1-\theta_0)$ does so for all goods, x is total expenditure or income, and n number of persons in the household. As with their equivalence scale model, θ_0 is a weighted average of the θ_i 's. n is the only demographic measure, though this can be modified to take the composition effect into account. Given unchanged PCE, the general scale effect, with $\theta_0 < 1$, allows a larger household to enjoy a higher standard of living than a smaller one, suggesting its effect is similar to a change in quality. So an estimate for θ_0 is obtained from a unit-value equation; the i th Engel curve is estimated subject to $\hat{\theta}_0$, providing a commodity-specific measure of the scale, $\hat{\theta}_i$, for each i . However, (Cramer 1969) first pointed out the lack of identification of the P-H scales. Once the adding-up restriction is imposed, the model breaks down. The reason for this is that we need n equations to estimate the m_i 's, but due to the budget constraint equation, we have only $(n-1)$ Engel curves for this purpose. Therefore, the model falls short by one equation: the ratios of specific scales are identified, but not their absolute values, see (Muellbauer 1980). Note, however, that the crucial identification problem with the P-H equivalence scale model poses an insurmountable hurdle here too. In particular, it is not possible to estimate the Engel economies of scales, which requires the prior identification of θ_0 , unless it is assumed that θ_i for food is equal to θ_0 , or unless some alternative identifying assumption is made. The Engel scale can therefore be seen as a special case of the P-H scale, with θ_i 's equal across all goods.

some simple cases.

Think of the P-H model as the Barten without substitution. Good 1 is food; good 2 is non-food.

$$q_i = m_i f_i (y / m_0)$$

where i is the commodity i scale, m_0 is the scale for income y (or weighted average of m_1 and m_2).

Here the food share is

$$w_1 = p_1 m_1 f_1 (y / m_0) / y = p_1 (m_1 / m_0) f_1 (y / m_0) / (y / m_0) = (m_1 / m_0) g_1 (y / m_0)$$

Now, suppose there are economies of scale in non-food but none for food. Then, as size expands with $y /$ size held constant, y / m_0 rises, since m_0 rises less than in proportion to size s . One would expect food expenditure *per capita* q_1 / s to rise since m_1 will be proportional to size s and, with y / m_0 higher and food a normal good, $f_1 ()$ will rise. D-P show that q_1 / s actually falls in empirical practice, where s stands for size with y / s is held constant and size increases.

If q_1 / s actually falls, then the food share

$$w_1 = p_1 q_1 / y = (p_1 q_1 / s) / (y / s)$$

clearly also falls, since y / s is constant by assumption.

Consider some cases:

(A) Suppose economies of scale affect food more than non-food. This is the opposite of D-P's assumption. Then as s rises with y / s constant, $f_1 (y / m_0)$ will rise. But m_1 will rise less than in proportion:

$$\frac{q_1}{s} = \frac{m_1}{s} f_1 (y / m_0)$$

So if m_1 / s falls proportionally more than $f_1 ()$ rises, q_1 / s will fall, as D-P say is true in practice.

More generally, consider

$$\frac{\partial \log(q_1 / s)}{\partial \log s} = \frac{\partial}{\partial \log s} [(\log m_1 - \log s) - \log f_1 (y / m_0)], \text{ with } y / s \text{ constant}$$

$$= \frac{\partial \log m_1}{\partial \log s} - 1 + \frac{\partial \log f_1}{\partial \log s}, \text{ with } y/s \text{ constant}$$

$$\log f_1 = g((\log y - \log s) - (\log m_0 - \log s)) \text{ so}$$

$$\frac{\partial \log f_1}{\partial \log s} = -\eta \left(\frac{\partial \log m_0}{\partial \log s} - 1 \right) = \eta \left(1 - \frac{\partial \log m_0}{\partial \log s} \right)$$

where η = income elasticity for food, for instance 0.4;

$$\frac{\partial \log m_0}{\partial \log s} = w \frac{\partial \log m_1}{\partial \log s} + (1-w) \frac{\partial \log m_2}{\partial \log s}$$

where w = food share.

For example, suppose

$$w=0.6, \frac{\partial \log m_1}{\partial \log s} = 0.7, \frac{\partial \log m_2}{\partial \log s} = 0.9$$

Then

$$\frac{\partial \log(q_1/s)}{\partial \log s} = 0.7 - 1 + 0.4(1 - 0.6 \times 0.7 - 0.4 \times 0.9) = -0.3 + 0.4 \times 0.22 = -0.212.$$

Here one easily gets the negative effect of size on food consumption per head.

(B) Suppose the same economies of scale for both food and non-food, e.g.

$$\frac{\partial \log m_1}{\partial \log s} = \frac{\partial \log m_2}{\partial \log s} = 0.7$$

Then

$$\frac{\partial \log(q_1/s)}{\partial \log s} = -0.3 + 0.4 \times 0.3 = -0.18$$

Effectively, this boils down to the Engel model. And again, one easily gets the negative effect of size on food consumption per head.

(C) Suppose economies to be higher for non-food: even then, it is *possible* to

$$\text{get D-P result, e.g. } \frac{\partial \log m_1}{\partial \log s} = 0.9, \frac{\partial \log m_2}{\partial \log s} = 0.7$$

$$\text{Then } \frac{\partial \log(q_1/s)}{\partial \log s} = (0.9 - 1) + 0.4(1 - 0.6 \times 0.9 - 0.4 \times 0.7) = -0.1 + 0.4 \times 0.18 = -0.04$$

This is negative again.

The equivalence scale, w.r.t., s is given by $0.6^{0.9} + 0.4^{0.7}$. However, this could not be deduced from the data. An elasticity of $\frac{\partial \log(q_1/s)}{\partial \log s} = -0.04$ is also

consistent with the Engel model where $\frac{\partial \log m_1}{\partial \log s} = \frac{\partial \log m_2}{\partial \log s} = 0.93333$. Finally,

if substitution is allowed, as in Barten's model, the negative direction of the size effect is reinforced. Suppose case (C): as s rises, m_1 rises relative to m_2 , so demand for food per capita drops a little more since food is now relatively more "expensive". It is then even easier to obtain D-P's negative size effect.

Note that case (C) can also explain the D-P difference in the size effect between developed and developing countries: reduce w_1 to 0.3, then $\frac{\partial \log(q_1/s)}{\partial \log s} = -0.004$, i.e. a less negative size effect¹⁰.

Some of the above issues are implicit in D-P, for instance their rejection of the second Engel 'law', see (Deaton and Paxson 2003) response to (Gan and Vernon 2003). Furthermore, D-P offer a list of potential explanations for their negative size effect, the first three of which, especially that based on direct food economies, do take into account the effect of private economies. However, they reject all such explanations as unlikely to be empirically important enough to resolve the D-P paradox.

If the P-H model is true, identifying assumptions are required to make welfare comparison across households of different size and composition, though, given price variation, the problem is, in principle, solvable for the Barten model. With no substitution permitted, no amount of extra price information can identify the P-H, or the Engel, scales, see (Muellbauer 1977 &

¹⁰Cases A and B are less likely than C. However, they suggest a negative, not a positive, size effect should be the expected outcome under a wide variety of assumptions about the scope for economies. The importance of case B is, of course, in making the hidden identifying assumption of Engel's model explicit.

1980). The basic issue that emerges from the above discussion is that to make welfare comparison across households, one needs to make identifying assumption in order to be able to estimate the P-H, or Engel scales; such assumptions cannot be empirically tested. To this extent, D-P have a case in questioning welfare interpretations of food share equations.

III. Conclusion

The effect of size expansion on the food share with constant per capita total expenditure remains negative. D-P regard this outcome, in the context of a (non-food) public goods model of economies of scale, as a 'paradox'. Such a conclusion is based on the implicit assumption that scope for direct economies in food consumption is negligible. This is unlikely. Even a moderate degree of economies in food consumption can explain the D-P negative size effect. Such possible explanations are mentioned in D-P, who, in addition, provide a few pieces of suggestive evidence in their support. Perhaps D-P dismiss some of their suggested explanations too readily. Such explanations can resolve the D-P 'paradox' if they are further examined.

References:

- Cramer, Jan. 1969. *Empirical Econometrics*. Amsterdam: North-Holland.
- David, Martin Heidenhain. 1962. *Family Composition and Consumption*. Amsterdam: North-Holland.
- Deaton, Angus. 1997. *The Analysis of Household Surveys, A Microeconomic Approach to Development Policy*. London: Johns Hopkins University Press.
- Deaton, Angus and Christina Paxson. 1998. "Economy of Scale, household size, and the demand for food." *Journal of Political Economy*. **106(5)**: 897-930.
- Deaton, Angus and Christina Paxson. 2003. "Engel's What? A Response to Gan and Vernon," *Journal of Political Economy*. **3(6)**: 1378-1381.
- Gan, Li and Victoria Vernon. 2003. "Testing the Barten Model of Economies of Scale in Household Consumption: Toward Resolving a Paradox of Deaton and Paxson." *Journal of Political Economy*. **3(6)**: 1361-1377.
- Koochi-Kamali, Feridoon. 2005, *Welfare and Consumption Rationing: A Study in Behaviour Based on a Wartime Iranian Household Expenditure Survey*, Unpublished D. Phil thesis, Oxford University
- Muellbauer, John. 1977. "Testing the Barten Model of Household Composition Effects and the Cost of Children." *Economic Journal*, **87(Sept.)**: 460-487.
- Muellbauer, John. 1980. "The Estimation of Prais-Houthakker Model of Equivalence Scales." *Econometrica*, **48(1)**: 153-176.
- Nelson, Julie. 1988. "Household Economies of Scale in Consumption: Theory and Evidence." *Econometrica*, **56(6)**: 1301-1314.
- Pollak Robert and Terence Wales. 1981. "Demographic Variables in Demand Analysis." *Econometrica*, **49(Nov.)**: 1533-1551.
- Prais, Sig. and Hendrik Houthakker. 1955. *The Analysis of Family Budgets*, Cambridge: Cambridge University Press, 2nd ed. 1971.

